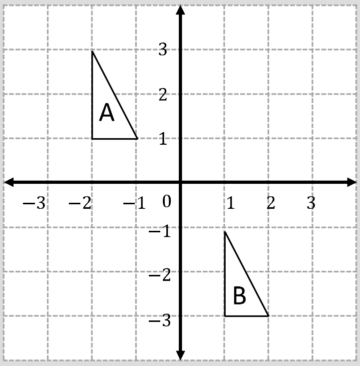
Mathematics Mastery
Maths Pack

Exercises

Week 1: Transformations

Session 1: Translation

Concept Corner

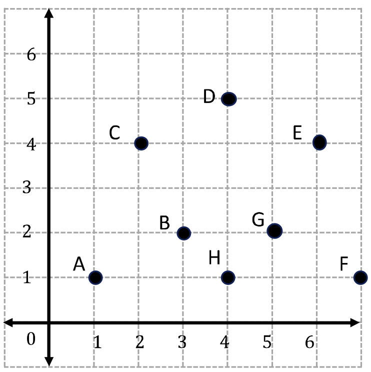


3	translated
4	direction
$\begin{pmatrix} 3 \\ -4 \end{pmatrix}$	vector

We say a shape has been ___________ when every point in the shape moves by the same distance in the same _____________. We can use a _______ to describe the translation.

e.g. The transformation from A to B is a translation by the vector ____. The shape moves ______ to the right and ______ down.

1.



a) Write down the coordinates of the points A – H

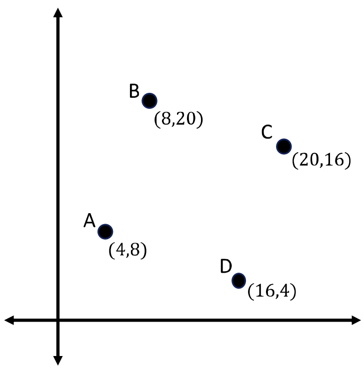
b) Describe the journey using vector notation:

e.g. A to B: $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

i) C to D ii) D to E iii) F to A

iv) D to C v) E to D vi) A to F

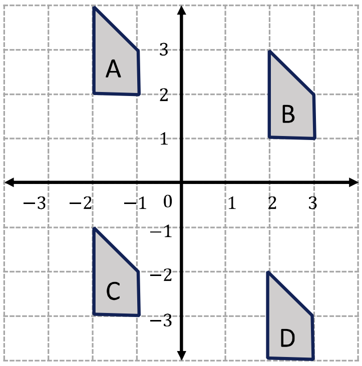
2. Describe the journey using vector notation:



a) A to B b) D to C c) D to A

d) C to B e) D to B f) A to C

3. Describe the transformation in each case:



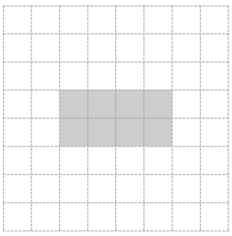
a) A to B b) B to D c) A to D

d) A to C e) C to D f) D to A

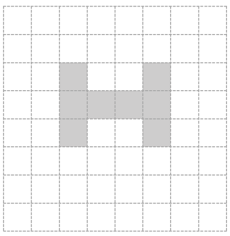
4. Sketch, on a single diagram, the outcome of translating the shape by:

$$\begin{pmatrix}2\\-2\end{pmatrix}, \quad \begin{pmatrix}-2\\-2\end{pmatrix}, \quad \begin{pmatrix}-2\\2\end{pmatrix}, \quad \begin{pmatrix}2\\2\end{pmatrix}, \quad \begin{pmatrix}4\\0\end{pmatrix}, \quad \begin{pmatrix}-4\\0\end{pmatrix}$$

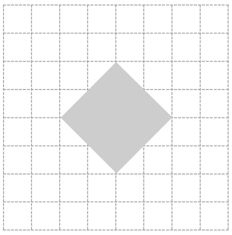
a)



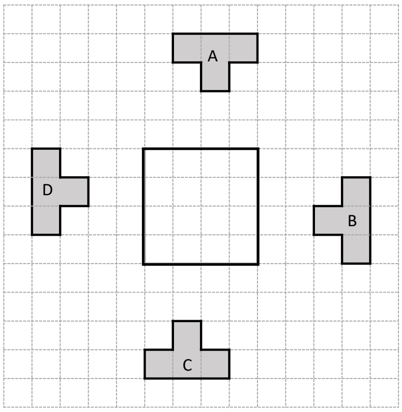
b)



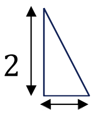
c)

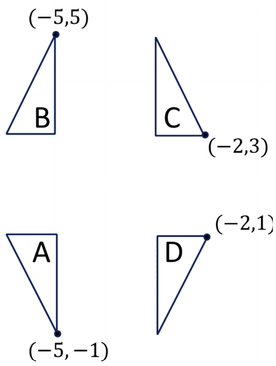


5. Describe four translations of the octagons that sends the shapes inside the square forming a tessellation pattern.



Questions for depth:

1. Four copies of the triangle  are arranged as follows:

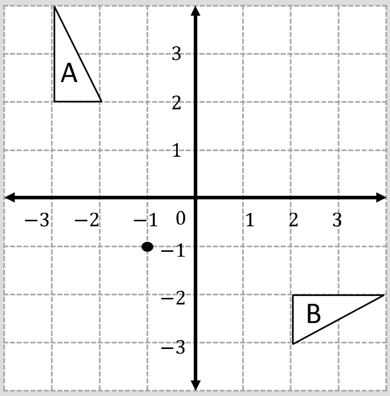


a) If A is translated by $\begin{pmatrix}3\\2\end{pmatrix}$, B is translated by $\begin{pmatrix}3\\-4\end{pmatrix}$ and C by $\begin{pmatrix}0\\-2\end{pmatrix}$ what quadrilateral is formed?

b) A is translated by $\begin{pmatrix}2\\0\end{pmatrix}$, B is translated by $\begin{pmatrix}2\\-2\end{pmatrix}$ and C by $\begin{pmatrix}0\\-2\end{pmatrix}$ what quadrilateral is formed?

Week 1 Session 2: Rotation

Concept Corner

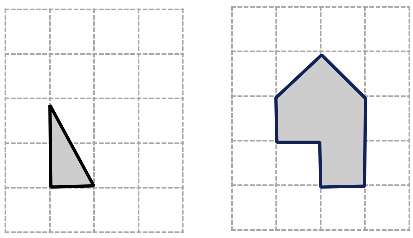


(−1,1)	rotate
90°	rotation
90°	anticlockwise

We can ___________a shape about a point called the centre of ___________ .

e.g. The transformation from A to B can be described as a rotation of _______ clockwise about the point ______ . The transformation from B to A can be described as a rotation of ______ _______________ about the same point.

1. Draw a copy of each shape after:



a) a 90° rotation clockwise

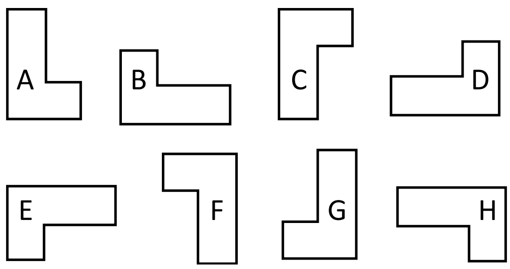
b) a 180° rotation

c) a 270° rotation clockwise

d) a 90° rotation anticlockwise

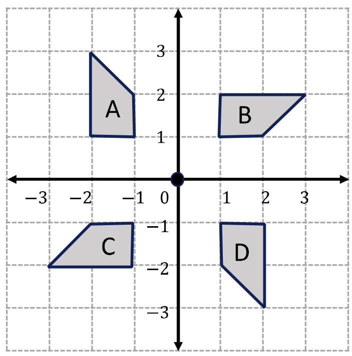
2. Generate **five statements** describing the angle and direction of rotation between two shapes:

   e.g. "H to C is a rotation of 270° clockwise."



3. Describe the following transformations:

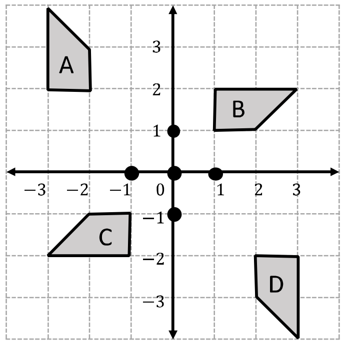
E.g. C to A "a rotation of 90° clockwise about the origin."



a) A to B   b) B to D   c) A to D

d) A to C   e) C to D   f) D to A

4. Describe the following transformations:

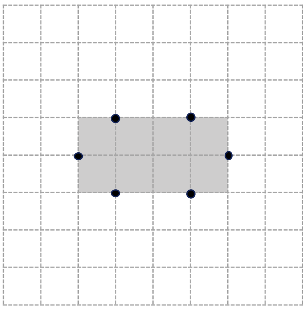


a) A to B b) B to D c) A to D

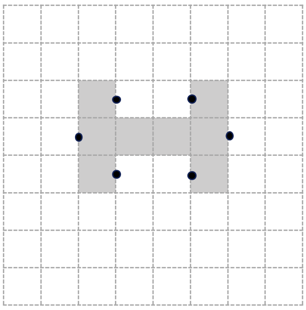
d) A to C e) C to D f) D to A

5. Sketch, on a single diagram, the outcome of rotating the shape by 180° about each of the six points:

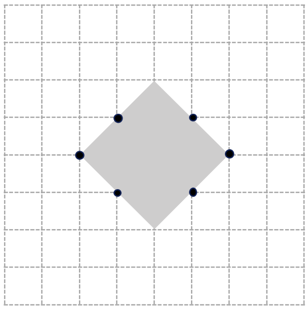
a)



b)

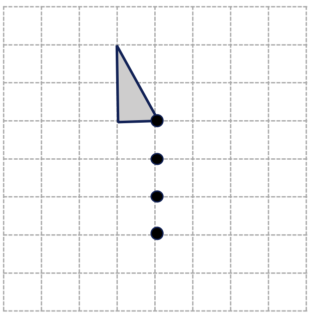


c)

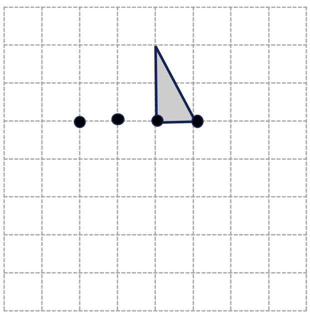


6. Sketch, on a single diagram, the outcome of rotating the shape by 90° clockwise about each of the points:

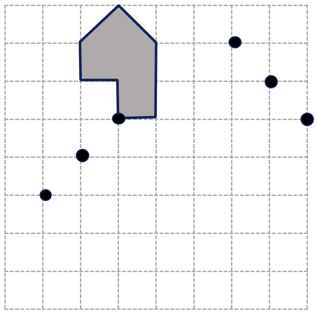
a)



b)

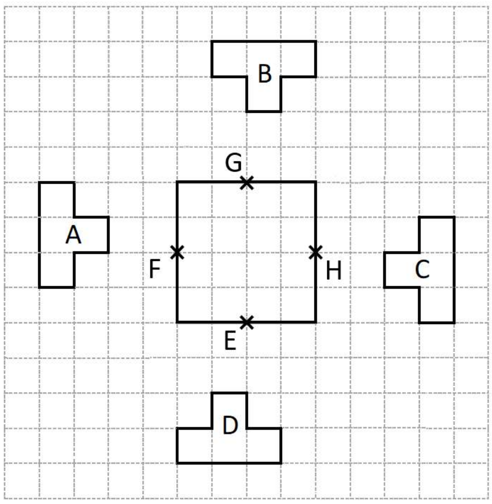


c)



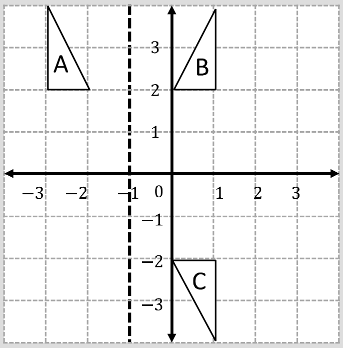
Questions for depth:

1. Describe four rotations of the octagons that sends the shapes inside the square forming a tessellation pattern.



Week 1 Session 3: Reflection

Concept Corner

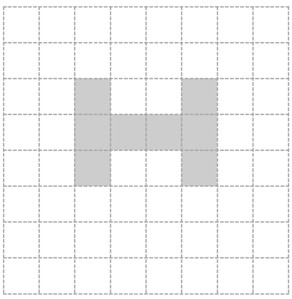
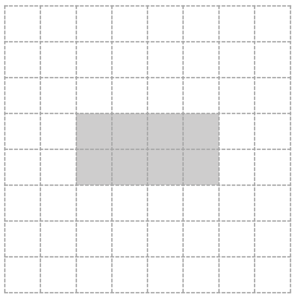
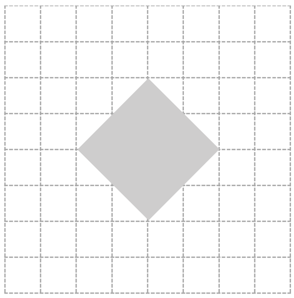


$x = -1$	reflect
equidistant	
$y = 0$	line

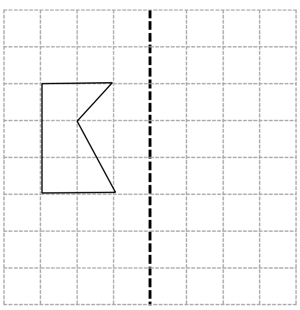
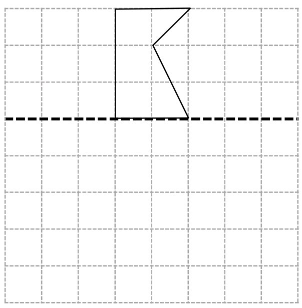
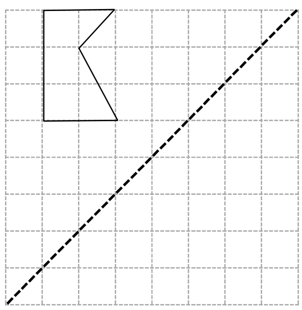
We can ___________a shape in a __________. A point and it's reflection are ___________ from the line of reflection.

e.g. The transformation from A to B can be described as a reflection in the line _____.
The transformation from B to C can be described as a reflection in the line _____.

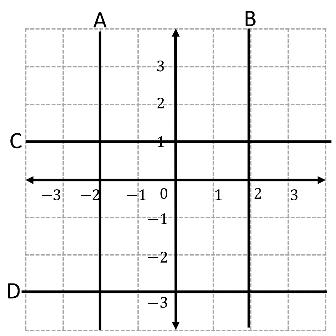
1. Identify the number of lines of symmetry :

a)  b)  c) 

2. Copy the shape and reflect it in the dotted line.

a)  b)  c) 

3. Write the equation for each line:



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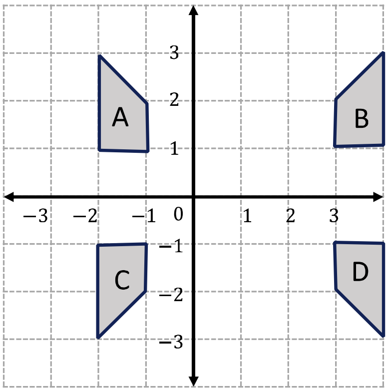
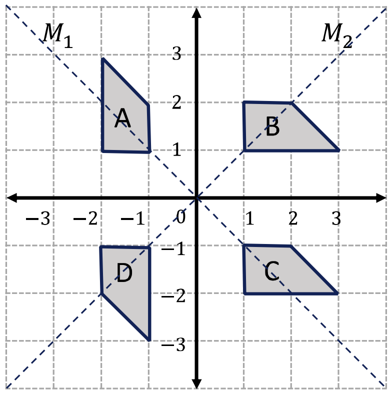
4. Describe the following transformations:



a) A to B b) B to D c) A to D

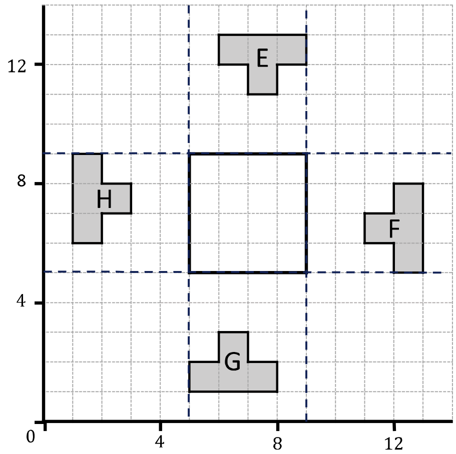
d) A to C e) C to D f) D to A

5. Two lines M_1 and M_2 are shown. Describe the following transformations:

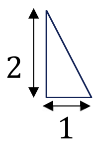


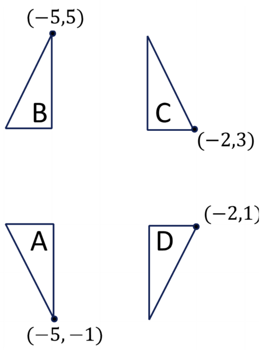
i) B to D ii) D to B iii) A to C

6. Describe four reflections of the octagons that sends the shapes inside the square forming a tessellation pattern.



Questions for depth:

1. Four copies of the triangle  are arranged as follows:



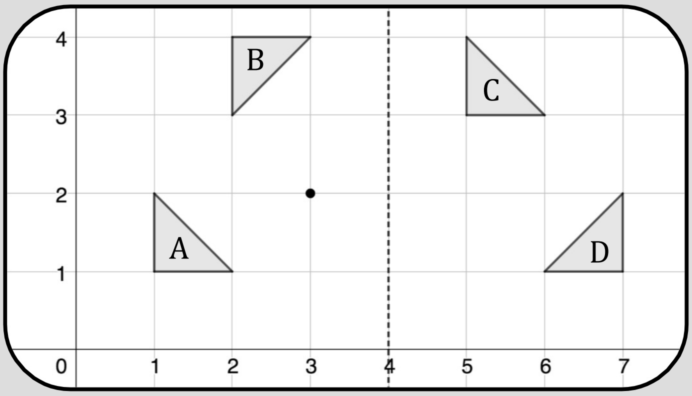
a) Describe the transformation from B to C

b) Describe the transformation from A to D

c) Describe the transformation from A to C

Week 1 Session 4: Isometries

Concept Corner



4	B	rotated
C	translated	
D	reflected	

Reflection, rotation and translation are all examples of isometries.

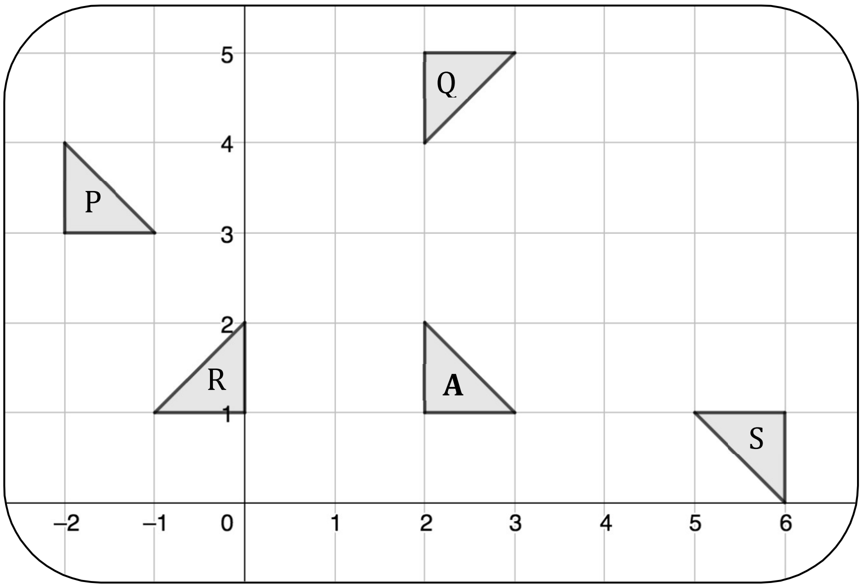
Triangle A has been _____________90° clockwise about (3, 2) to give triangle ____

Triangle A has been ____________ in the line $x =$ ____ to give triangle ____

Triangle A has been ____________ by $\binom{-4}{-2}$ to give triangle ____

1. Look at the image below. Write down **single transformations** to which could transform:

a) A onto R
b) A onto P
c) A onto Q
d) A onto S

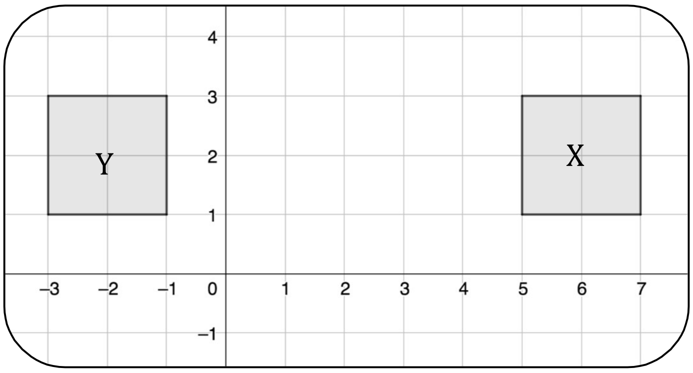


2. Look at the triangles in Q1.
a) What single transformation could transform Q onto S?
b) What single transformation could transform R onto Q?

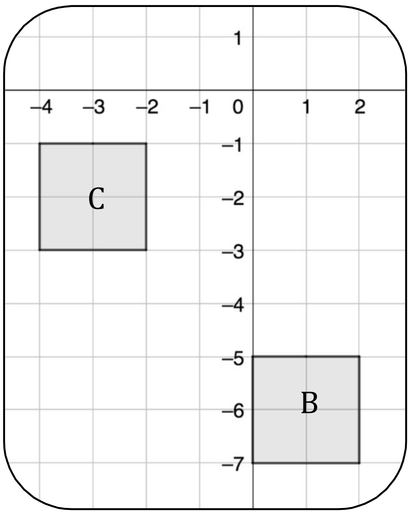
3. In your book draw a set of axes where both x- and y-axes range from 0-10.
Copy triangle A from Q1 onto your axes.
Now, on your axes, draw the images created by the following transformations:

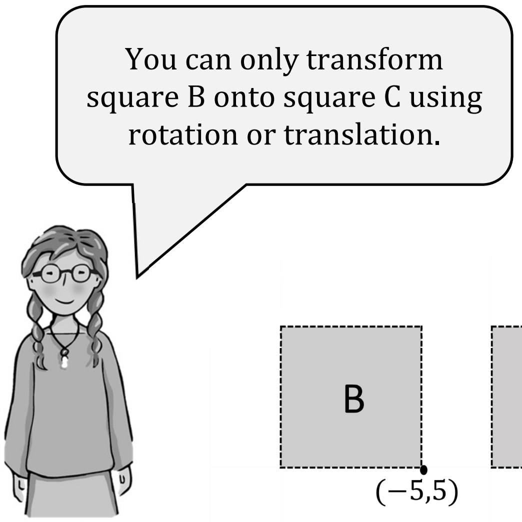
a) Translate A by $\binom{5}{-1}$
b) Reflect A in the line $y = 5$
c) Rotate A $90°$ clockwise about $(7, 1)$

4. Look at the diagram below. Write down **three different** transformations that could have transformed square X onto square Y.



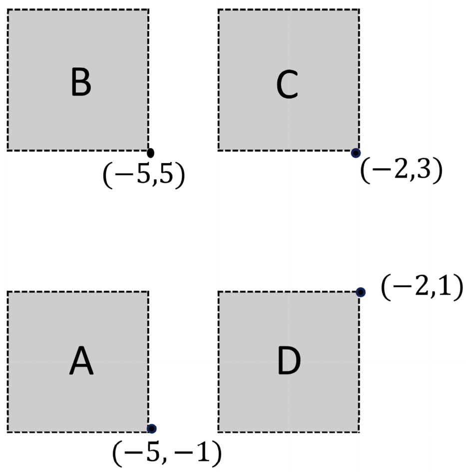
5. Look at the image below. Do you agree with Rosie's statement? Explain your answer.





Questions for depth:

1. Explore the different transformations that takes one square to another in the following image:



Week 2: More transformations

Session 1: Combining reflections

Concept Corner

A combination of ______________ can sometimes be described by a different single transformation.

e.g. Triangle A is reflected in the line M_1 to give ___.

A' is then __________ in the line M_2 to give ____.

A could also be mapped to A'' by a translation by the vector _____.

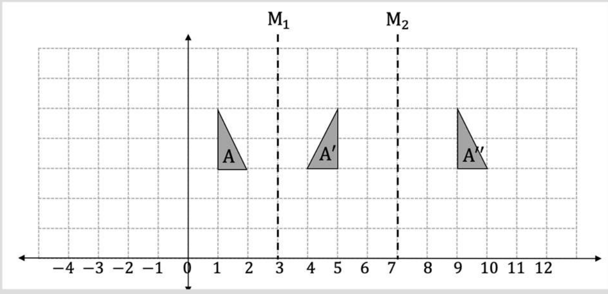
transformations

reflected

$\begin{pmatrix} 8 \\ 0 \end{pmatrix}$

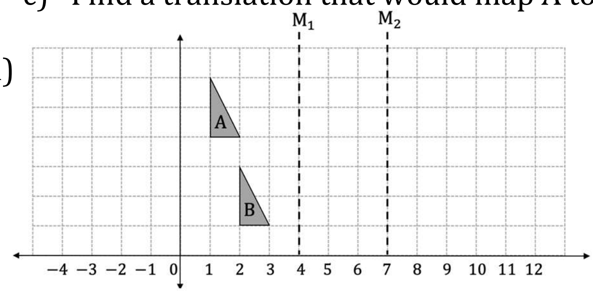
A'

A''

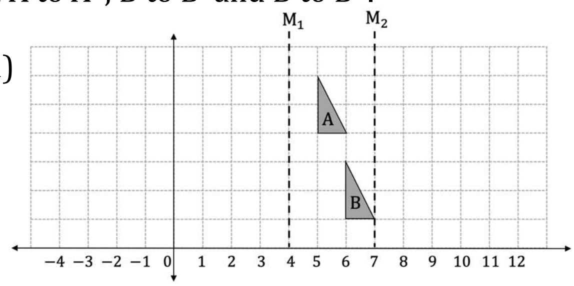


1. For each question:
   a) Reflect A and B in line M_1 followed by M_2. Label the results A' and B'.
   b) Reflect A and B in line M_2 followed by M_1. Label the results A'' and B''.
   c) Find a translation that would map A to A', A to A'', B to B' and B to B''.

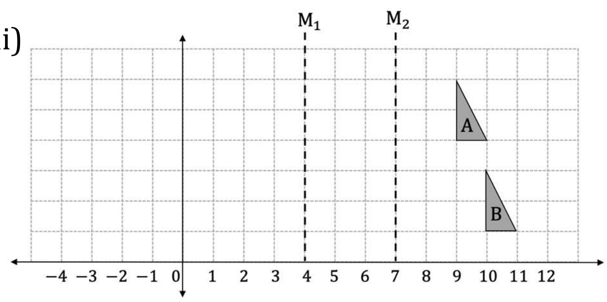
i)



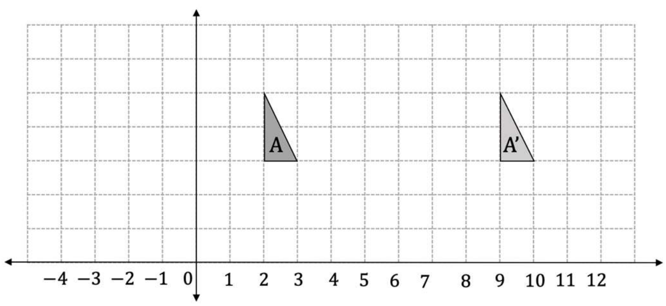
ii)



ii)



2. a) Give the translation that maps A to A'?
   b) Find the equation of two possible mirror lines that would reflect triangle A to A'.
   c) Find another three examples of two mirror lines that would map A to A'. What do all the pairs have in common?



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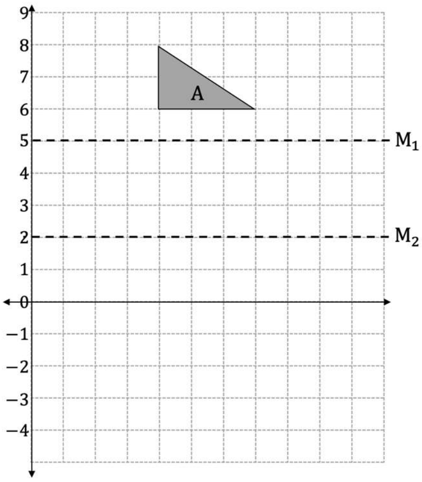
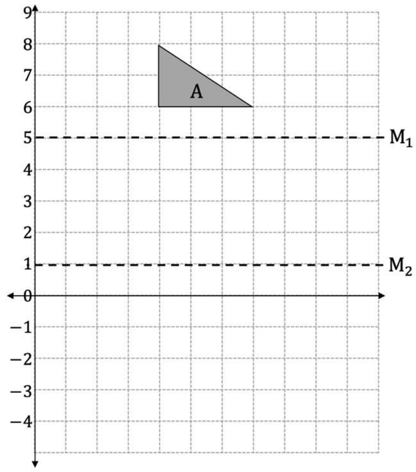
3. For each question:
   a) Reflect A line M_1 followed by M_2. Label the results A’.
   b) Find a translation that would map A to A’.
   c) Describe the relationship between this translation and the lines M_1 and M_2.

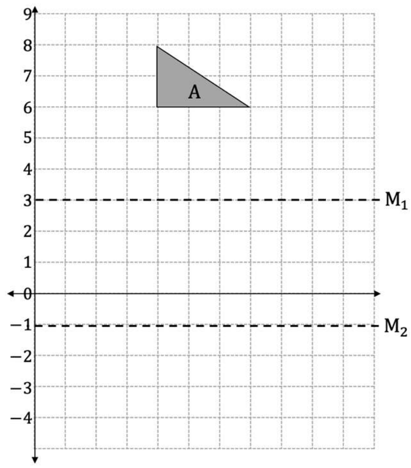
i)



ii)



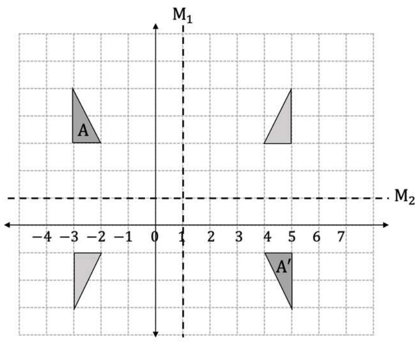
iii)

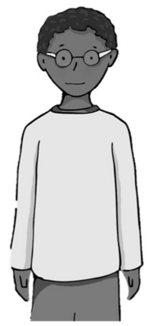


4. Sam tries reflecting the triangle A in a vertical and then a horizontal line. He notices that if he reflects in these lines in either order, the triangle ends up in the same position.

   How is this different to when the two lines are parallel?

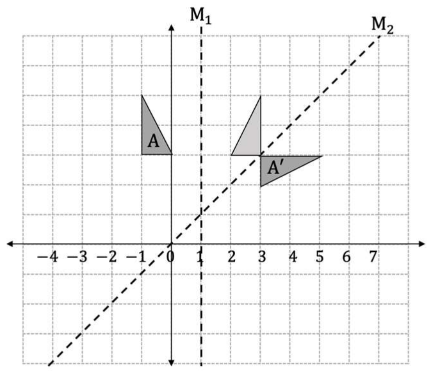
   Does this always work if the two lines of reflection are perpendicular?





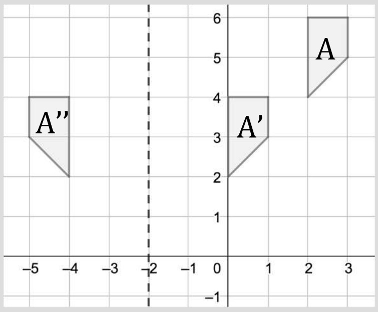
Questions for depth:

1.
   a) Reflect A line M_1 followed by M_2. Label the results
   b) What transformation would map A to A’
   c) Try moving A to a different position. What do you notice?
   d) How does this transformation relate to the lines M_1 and M_2.



Week 2 Session 2: Combining translations and reflections

Concept Corner



transformation

$\begin{pmatrix}-2\\-2\end{pmatrix}$

reflected

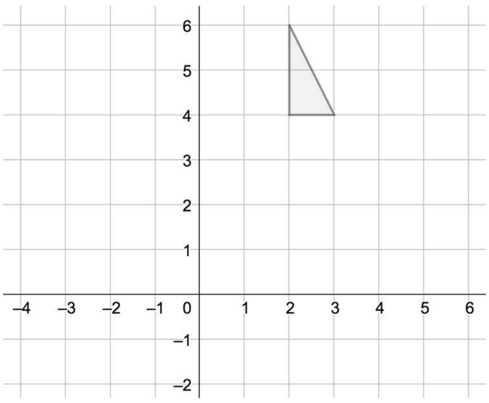
$x = -2$

We can combine a translation and a reflection by performing one ___________ after the other. For example: A is translated by the vector ____ to give A'. A' is then __________ in the line ______ to give A''.

1.

a) What are the co-ordinates of the vertices of this triangle?

b) State the co-ordinates of the vertices of the translated shape:



i) A translation by the vector $\begin{pmatrix}2\\0\end{pmatrix}$

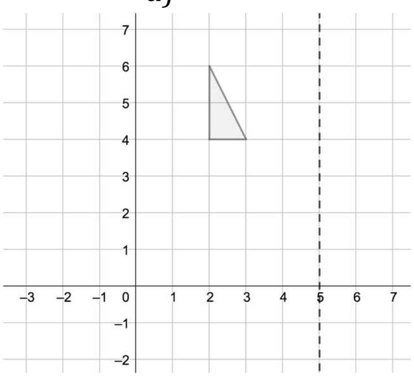
ii) A translation by the vector $\begin{pmatrix}0\\-3\end{pmatrix}$

iii) A translation by the vector $\begin{pmatrix}1\\1\end{pmatrix}$

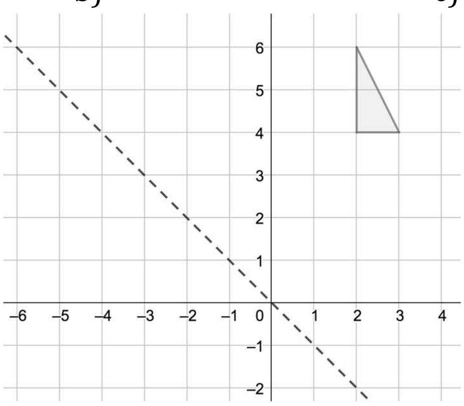
iv) A translation by the vector $\begin{pmatrix}-2\\2\end{pmatrix}$

2. State the co-ordinates of the reflected triangle:

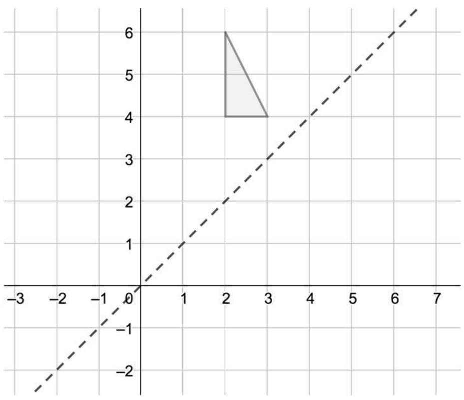
a)



b)

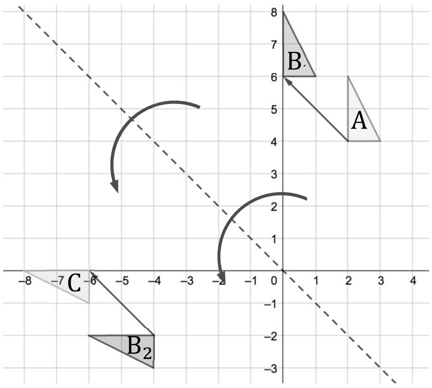


c)



d) A reflection of the triangle in the line $y = 3$

3. A student is exploring the effect of combining a translation and a reflection.



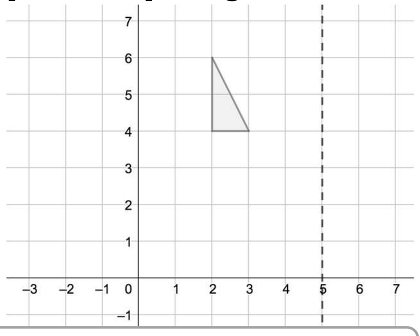
Reflect in the dotted line

Translate by the vector $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

The order in which I reflect and translate doesn't matter in this case.
$A \rightarrow B_1 \rightarrow C$ or
$A \rightarrow B_2 \rightarrow C$

4. Explore completing the two transformations in different orders:

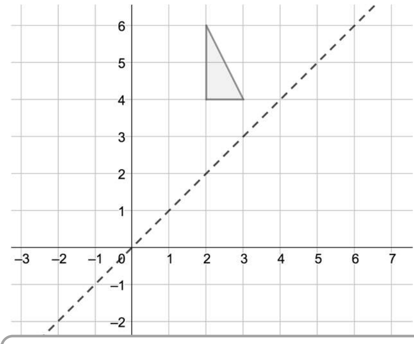
a)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

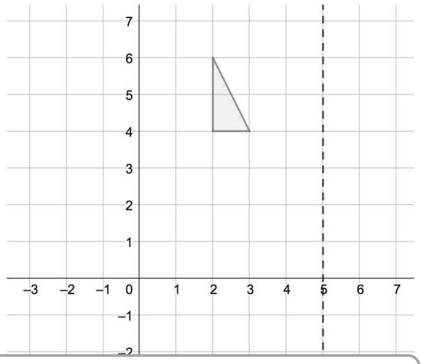
b)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

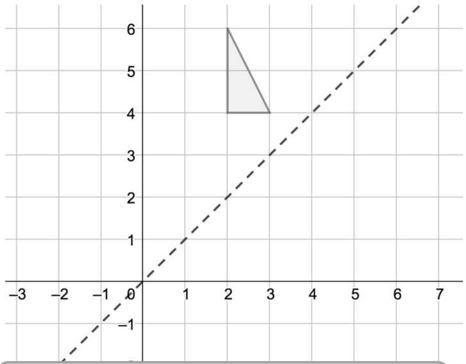
c)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

d)



Reflect in the dotted line

Translate by the vector $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

Questions for depth:

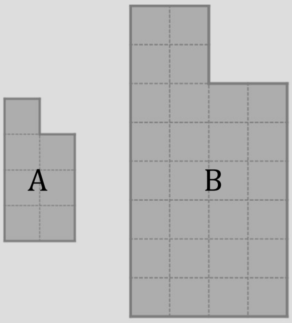
1. How far apart are the two options?

Option 1: A shape is translated by a vector $\begin{pmatrix} a \\ 0 \end{pmatrix}$ then reflected in the line $x = b$

Option 2: The same shape is reflected in the line $x = b$ then translated by a vector $\begin{pmatrix} a \\ 0 \end{pmatrix}$

Week 2 Session 3: Enlargement

Concept Corner

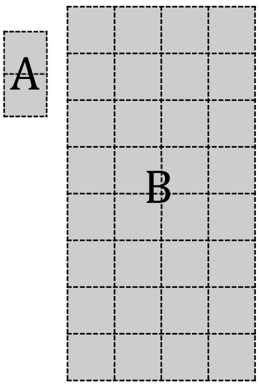


$\frac{1}{2}$ | scale factor | enlargement | 2 | half

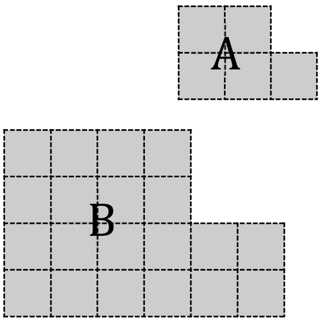
We can enlarge shapes using a ____________ ___________. The enlarged shape will have the same ‘form’ as the original shape. For example the transformation from A to B is an __________ of scale factor ____, the sides of the polygon are twice as long. The transformation from B to A is an enlargement of scale factor ____, the sides are ______ as long.

1.

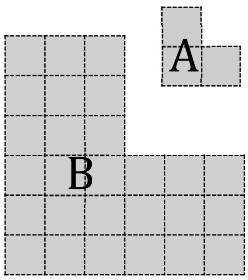
a)



b)



c)



i) State the scale factor of enlargement for each of the following transformations from A to B

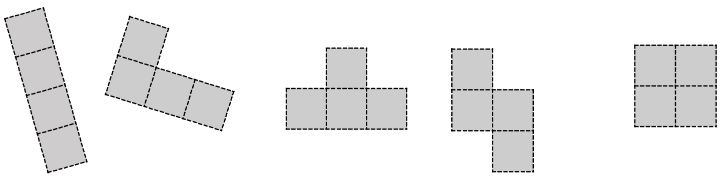
ii) State the scale factor of enlargement from B to A

2. On squared paper draw an enlargement of each shape:

a) with a scale factor of 2

b) with a scale factor of 3

c) with a scale factor of 1

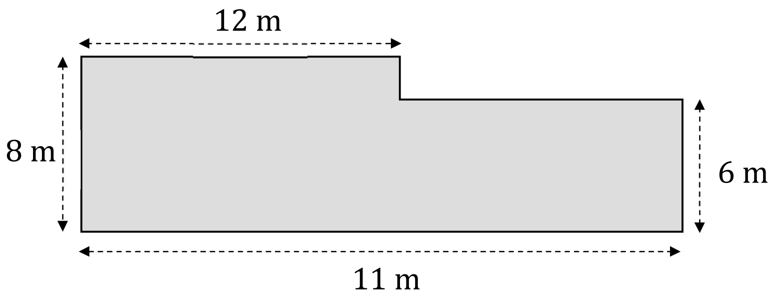


3. Find the perimeters of each of the shapes in Q1. What do you notice?

4.

Draw a **sketch** of the following shape after it has been enlarged by a scale factor of :

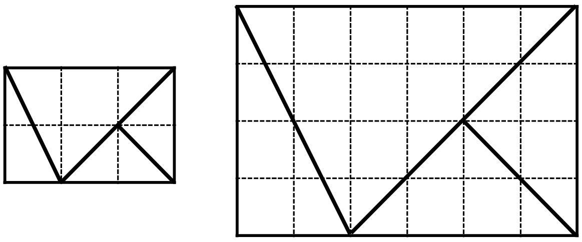
a) 2 b) $\frac{1}{2}$ c) 3



5. Find the perimeters of each of the enlarged shapes in Q4.

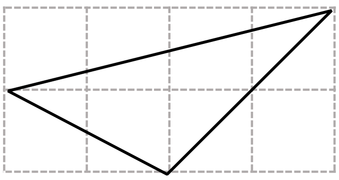
6.

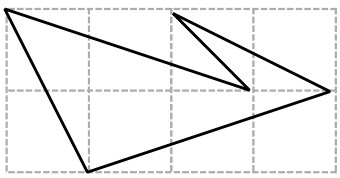
a) The 2 by 3 rectangle below was partitioned into 4 triangles and enlarged by a scale factor of 2:

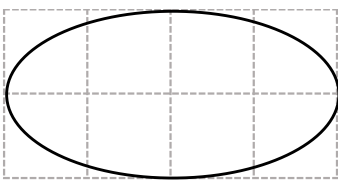


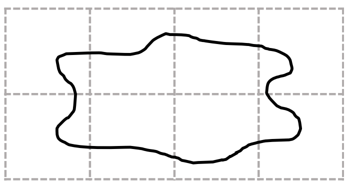
Draw a 2 by 3 rectangle on squared paper and partition it your own way, then enlarge it by a scale factor of 2.

7. Sketch each of the following shapes on square paper and then enlarge them by a scale factor of 3 :

a) 

b) 

c) 

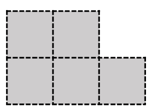
d) 

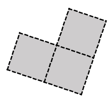
Questions for depth:

1. Hexagon A is enlarged by a scale factor of a and hexagon B by a scale factor of b. The perimeters of the two enlarged shapes are the same.

a) Suggest four possible values for a and b

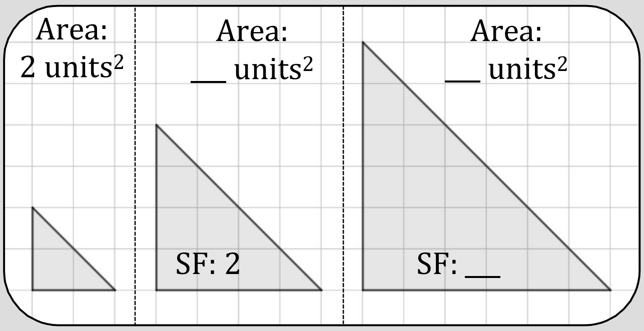
b) Write an equation linking a and b

A: 

B: 

Week 2 Session 4: Enlargements and area

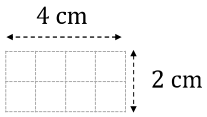
Concept Corner

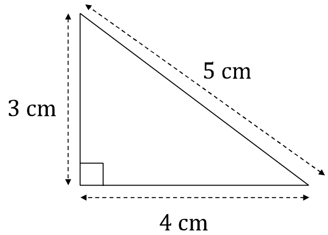


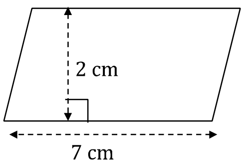
4 | 3 | changes | 8 | 18 | enlarge | scale factor

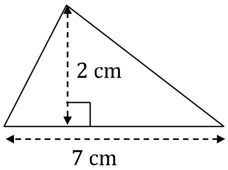
When we _______ a shape can affect the area. There is a relationship between the scale factor and how the area _______. The example above shows two different enlargements of the triangle on the left. If we enlarge by a ______ ______ of 2, the area becomes ___ times greater.

1. Find the area of the following shapes:

a) 

b) 

c) 

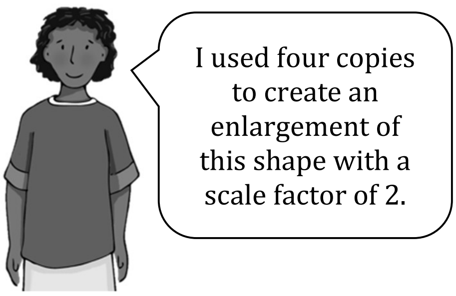
d) 

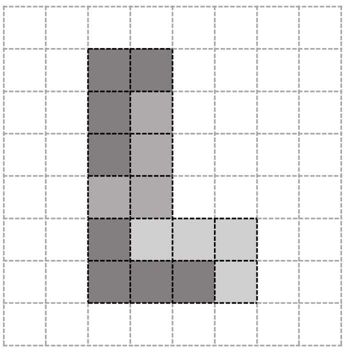
2.

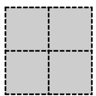
a) **Sketch** each of the shapes above following an enlargement of scale factor 3

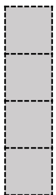
b) Find the area of the enlarged shapes, what do you notice?

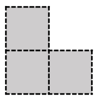
3. Hassan is trying to use copies of a shape to create an enlargement. Use four copies of each shape to enlarge them by a scale factor of 2.





a) 

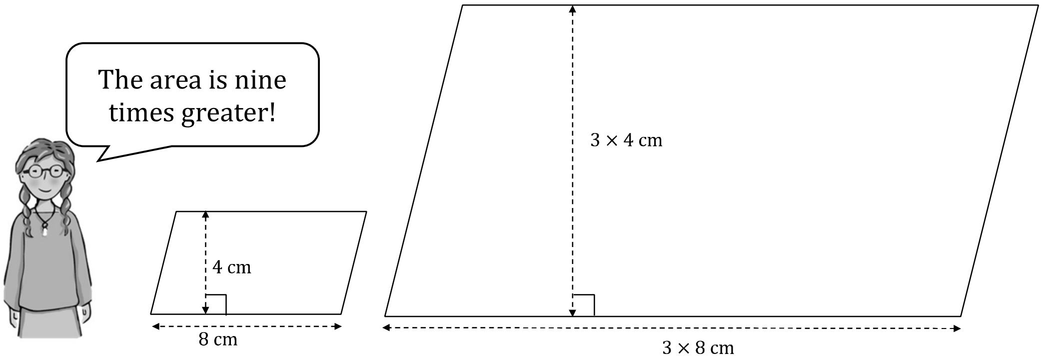
b) 

c) 

Mathematics Mastery

Ark

4. Rosie is comparing the area of a parallelogram before and after it has been enlarged:



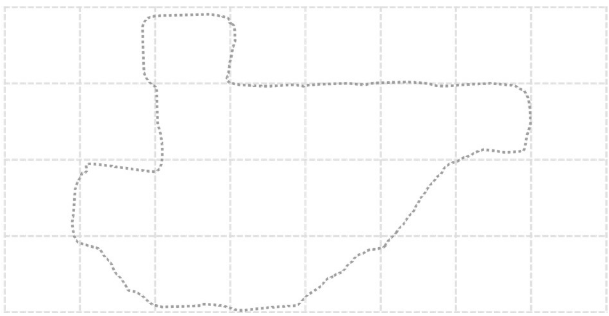
Area = $4 \times 8 \text{ cm}^2 = 32 \text{ cm}^2$

Area = $3 \times 4 \times 3 \times 8 \text{ cm}^2 = 9 \times 32 \text{ cm}^2$

Use a similar strategy to show many times greater the area is following an enlargement by scale factor:

a) 5 b) 12 c) n

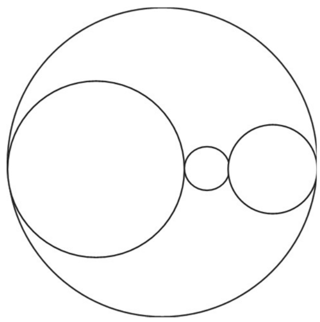
5. A map of an island is drawn on a square centimetre grid. The actual island is an enlargement of the map by a scale factor of approximately 2 000 000.



a) Estimate the length of the coast line.
b) Estimate the area of land of the island.

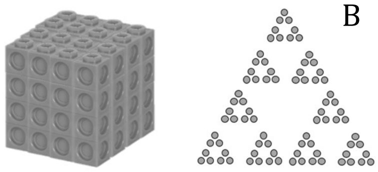
Questions for depth:

1. Three circles of radius $1\ cm$, $2\ cm\ and\ 6\ cm$ sit inside a larger circle. The four centres lie on the same horizontal line. Compare the area of the smallest circle to the three larger circles. How many times greater are they?



Week 3: Prime factorisation 1
Session 1: Indices

Concept Corner

A  B

4^3	index
3^4	cubed
4	squared

When numbers are written "*to the power of*" it is called ____________ notation.
So 5×5 can be written as 5^2, and is said '5 to the power of 2', or '5 ______________'.
$6 \times 6 \times 6 \times 6$ can be written as 6^4, and is called 6 to the power of ____.

Diagram A shows $4 \times 4 \times 4$ which can be written as ____ and is said '4 ____________'.
Diagram B shows $3 \times 3 \times 3 \times 3$ or ____.

1. Write the following calculations using index notation:

   a) 6×6

   b) $6 \times 6 \times 6 \times 6$

   c) $2 \times 2 \times 2 \times 2 \times 2$

2. Find the product of Q1 parts a) and b), write the answer in index notation.

3. Copy the equations below. Circle those that **are** true, and cross through those that aren't true.

$9^1 = 9$ $\qquad$ $5^3 = 25 \times 5$

$5^3 = 3 \times 3 \times 3 \times 3 \times 3$ $\qquad$ $10^2 = 2^{10}$

$8^3 = 24$ $\qquad$ $8^4 = 8 \times 8 \times 8 \times 8$

4. Place the correct symbol (<, >, or =) between each pair of numbers

   a) 2^3 _____ 3^2

   b) 2^4 _____ 4^2

   c) 3^3 _____ 5^2

   d) 1^8 _____ 1^5

5. Organise the diagrams into the following groups:

2 to the power of 4:

A ____ ____

3 to the power of 3:

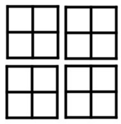
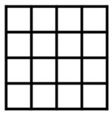
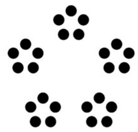
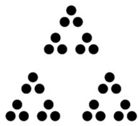
____ ____ ____

4 to the power of 2:

____ ____ ____

5 to the power of 2:

F ____ ____

2^4 A	4×4 B	 C	3^3 D
 E	5×5 F	4^2 G	 H
5^2 I	 J	 K	$2 \times 2 \times 2 \times 2$ L

6. Match the calculations on the left to the versions written in index form.

$2 \times 2 \times 7 \times 7 \times 7$	$5^3 \times 3$
$5 \times 3 \times 5 \times 3$	$2^2 \times 3^2 \times 5^2 \times 7$
$7 \times 7 \times 5 \times 5 \times 3 \times 3 \times 5 \times 7$	$3^2 \times 5^2$
$5 \times 5 \times 3 \times 3 \times 2 \times 2 \times 7$	$2^2 \times 7^3$
$5 \times 5 \times 5 \times 3$	$3^2 \times 5^3 \times 7^3$

7. $3^a > 100$. Find the smallest integer value of a.

8. 2^b is a square number. Find **three different** possible values for b.

Questions for depth:

1. Use the associative property of multiplication to write these calculations using index notation

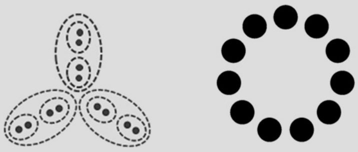
   a) 2 × 4

   b) 5 × 15

   c) 3 × 12

Week 3 Session 2: Prime factors

Concept Corner



5	can
12	cannot
11	prime

Prime numbers, like 5 and ____, can be multiplied together to make other numbers.

For example, 2s and 3s can be multiplied to make _____.

Prime numbers ___________ be made by multiplying smaller primes.

All non-prime numbers greater than 2 can be made by multiplying ____________ numbers, e.g. $70 = 2 \times \_\_\_\_ \times 7$.

1. Copy and complete the frames to find ways of showing the numbers as products of **different** combinations of factors.

a) $24 = \_\_ \times \_\_$

$24 = \_\_ \times \_\_ \times \_\_$

$24 = \_\_ \times \_\_ \times \_\_ \times \_\_$

b) $120 = \_\_ \times \_\_$

$120 = \_\_ \times \_\_ \times \_\_$

$120 = \_\_ \times \_\_ \times \_\_ \times \_\_$

2. Copy and complete so that each equation shows a number as the product of **prime** factors.

a) $12 = 2 \times 3 \times \_\_$

b) $20 = \_\_ \times 2 \times 5$

c) $30 = 2 \times \_\_ \times 5$

d) $36 = \_\_ \times 2 \times 3 \times 3$

e) $45 = 3 \times \_\_ \times \_\_$

f) $54 = 2 \times 3 \times \_\_ \times \_\_$

3. Rewrite the following products so that only prime factors are used

a) $24 = 2 \times 3 \times \mathbf{4}$

b) $48 = 2 \times \mathbf{24}$

c) $60 = \mathbf{4} \times \mathbf{15}$

d) $72 = \mathbf{4} \times 2 \times \mathbf{9}$

4. How many different products can you find by placing different combinations of the factor cards into the frame?

$\square \times \square \times \square =$

Factor cards

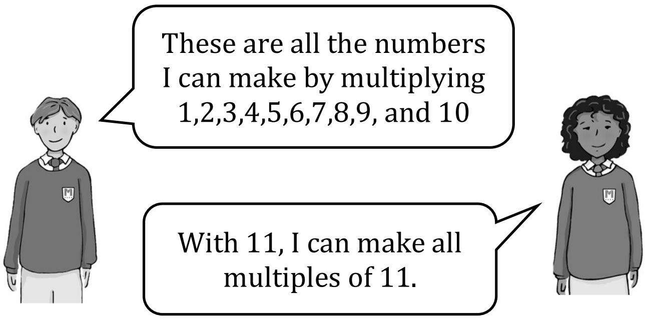
2 3 5

2 3

5. Gavin and Brenda have chosen a list of factors to multiply together.

Gavin has shaded in the numbers he thinks he can make by multiplying these factors (he can use factors more than once in each multiplication).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



a) Show how Gavin can make the following numbers by multiplying factors from his list:

$i)$ $28 =$ $\qquad$ $ii)$ $63 =$ $\qquad$ $iii)$ $42 =$

b) Do you agree that Brenda will be able to make all multiples of 11? Explain why or why not.

c) Gavin thinks he could remove factors from the list and **still make** all the shaded numbers.

Which factors could he remove? Why?

Questions for depth:

The grid has been shaded in the same way as in question 4) above, using a list of **seven** factors.

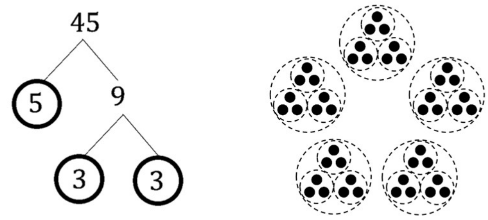
1. What was the list of factors?

2. What are the next three numbers greater than 100 that can be shaded?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Week 3 Session 3: Prime factorisation

Concept Corner



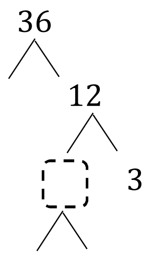
3 | compound | product | index

Numbers such as 18 and 45 are called ___________ integers and can be built as the ___________ of prime factors.

e.g. 45 can be written as _____ × 3 × 5, or 3^2 × 5 in ___________ notation.

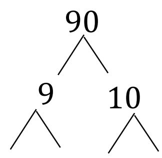
1. Copy and complete the prime factorisation trees and write the numbers as products of prime factors.

a)



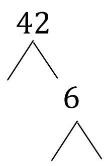
36 = __________

b)



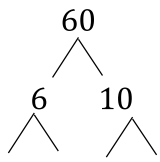
90 = __________

c)



42 = __________

d)



60 = __________

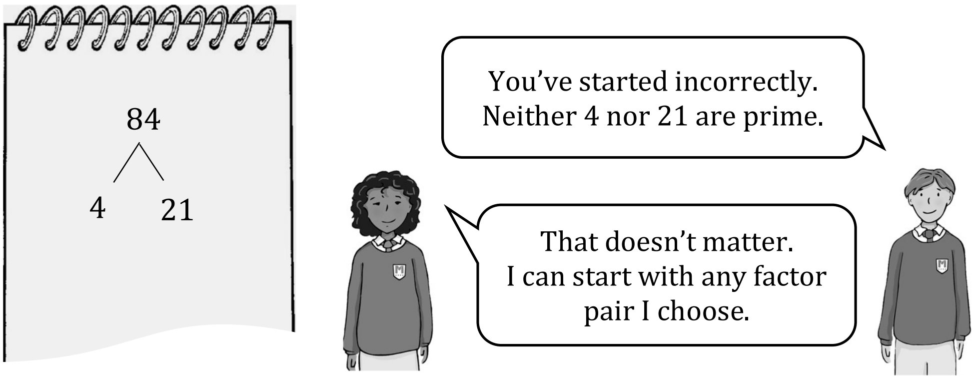
2. Write the following numbers as products of their prime factors.:

a) 72 b) 175

c) 144 d) 1750

e) 350 f) 216

3. Brenda is writing out the prime factor tree for 84. Gavin is commenting on her working.



Who do you agree with? Explain your answer. You may use examples to help.

4. Look at your answers to question 2. and answer the questions below.

a) Compare the prime factors of parts a), c) and f). What do you notice?

b) Compare the prime factors of parts b), d) and e). What do you notice?

c) Write down the prime factors of 720. Explain how you can use your answer to 2a) to help.

5. The cards below show three different numbers ***a***, ***b***, ***c*** and ***d*** written as products of their prime factors.
Decide if the statements below are **true or false.**

$a = 2 \times 3^2 \times 5^2$	$b = 2 \times 3^2 \times 5^3$
$c = 2^3 \times 3^2 \times 5^3$	$d = 2^2 \times 3^2 \times 5^2$

a) ***d*** is twice the value of ***a***

b) ***c*** is three times the value of ***b***

c) ***c*** is ten times the value of ***d***

d) ***b*** is less than ***d***

Questions for depth:

1. Two numbers, m and n have been written as the products of their primes, where x, y and z are **different** prime numbers. Decide whether the statements below are true or false.

a) ***m*** and ***n*** can be equal in value
b) ***m*** and ***n*** are square numbers
c) ***m*** and ***n*** are multiples of $(x^2 \times y^2 \times z^2)$

$m = x^2 \times y^3 \times z^4$
$n = x^4 \times y^3 \times z^2$

Week 3 Session 4: Using the prime factorisation

Concept Corner

Prime factorisations can be used to deduce factors of a number:

$$84 = \boxed{2^2 \times 3} \times \boxed{7} = 12 \times 7$$

12 | prime | 7 | pair | 21 | 4 | 21

In the example above the _________ factors 2, 2, and 3 have been multiplied together showing us that ____ and ____ are a factor ________ of 84.

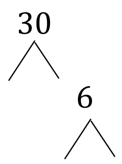
Using a similar approach another factor pair of 84 can be made:

$2 \times 2 =$ ____ $\quad\quad 3 \times 7 = 21$

So, 4 and ____ are a factor pair of 84.

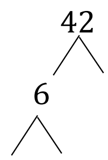
1. Copy and complete the prime factorisation trees and write the numbers as products of prime factors.

a)



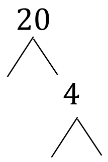
$30 =$ __________

b)



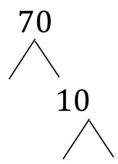
$42 =$ __________

c)



$20 =$ __________

d)



$70 =$ __________

2.

a) Use your answers to question 1 to help work out how many factors each of the numbers 30, 42, 20 and 70 has.

b) What's the same or different about the **number of factors** each number has? Explain why this is the case.

c) Find a number that has the same number of factors as 20.

3. The numbers in the box below have been written as the product of their prime factors. Use this information to help you answer the questions below.

72 $= 2^3 \times 3^2$	**75** $= 3 \times 5^2$
168 $= 2^3 \times 3 \times 7$	**30** $= 2 \times 3 \times 5$
245 $= 5 \times 7^2$	**42** $= 2 \times 3 \times 7$

a) 75 = 15 × _____

b) Does 168 have a factor that is a square number?

c) Two of the numbers multiply to make a square number, which two?

d) The product of all of the numbers in the box is 280052640000. Write 280052640000 as a product of prime factors

4. $2 \times 3^3 \times 5 = 270$

Every factor pair for 270 will have one even factor and one odd factor.

Do you agree or disagree with Nicola? Explain why.

5. $210 = 2 \times 3 \times 5 \times 7$ $\quad$ $220 = 2^2 \times 5 \times 11$ $\quad$ $230 = 2 \times 5 \times 23$

Which of these numbers has the most factors? Which of these numbers has the least factors?

6. Which of the numbers below has the **greatest** factor?

$315 = 3^2 \times 5 \times 7$ $\quad$ $210 = 2 \times 3 \times 5 \times 7$ $\quad$ $220 = 2^2 \times 5 \times 11$

Questions for depth:

1. Look at the number below written as the product of its prime factors.

$$42336 = 2^5 \times 3^3 \times 7^2$$

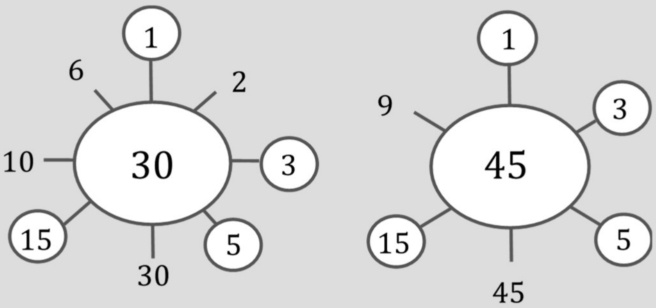
a) $42336 \div a$ results in an odd integer. What are the greatest and least possible values of a?

b) $42336 \times b$ results in a cube number. What are the greatest and least possible values of b?

Week 4: Prime factorisation 2

Session 1: Highest common factor

Concept Corner



15	HCF
factors	
common	

The highest common factor, abbreviated as the _____, can be found by listing the ____________ of each number.

We can use the diagram to see that the HCF of 30 and 45 is ____.

1. List all of the factors of:

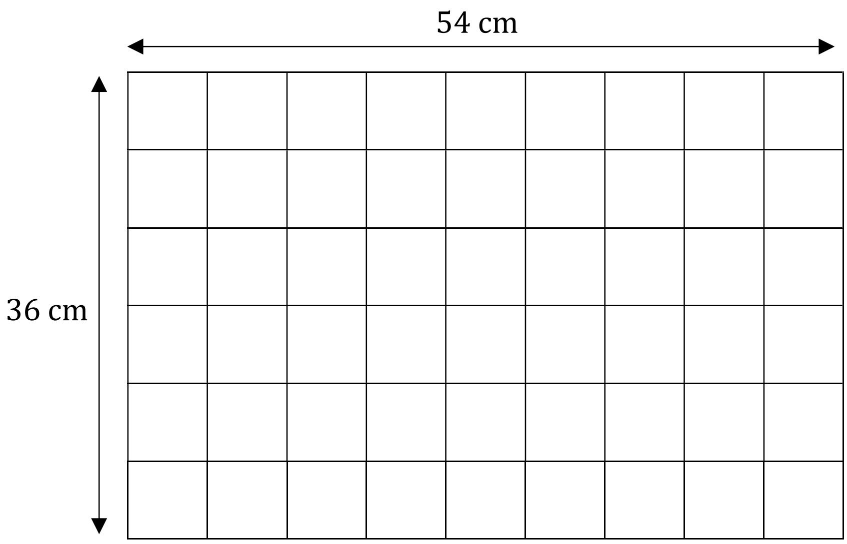
a) 28 b) 54 c) 36 d) 19

2. Identify **all the common factors** of:

a) 28 and 54 b) 28 and 36 c) 28 and 19 d) 54 and 36

3. Find the HCF for the pairs in Q2.

a) What size squares has this student split the rectangle in to?
b) What other squares size squares can you split it into?



4.

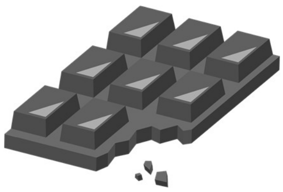
a) Find three pairs of numbers that have a HCF of 12.

b) What other factors do the pairs have in common?

5. Find the HCF of the following:

a) 12 and 18	b) 12 and 30	c) 12 and 42	d) 12 and 54
e) 16 and 24	f) 40 and 24	g) 64 and 24	h) 88 and 24

6. A shop sells boxes of chocolate. In total there are 252 dark chocolates and 180 milk chocolates. If every box is identical, how many boxes could there be?



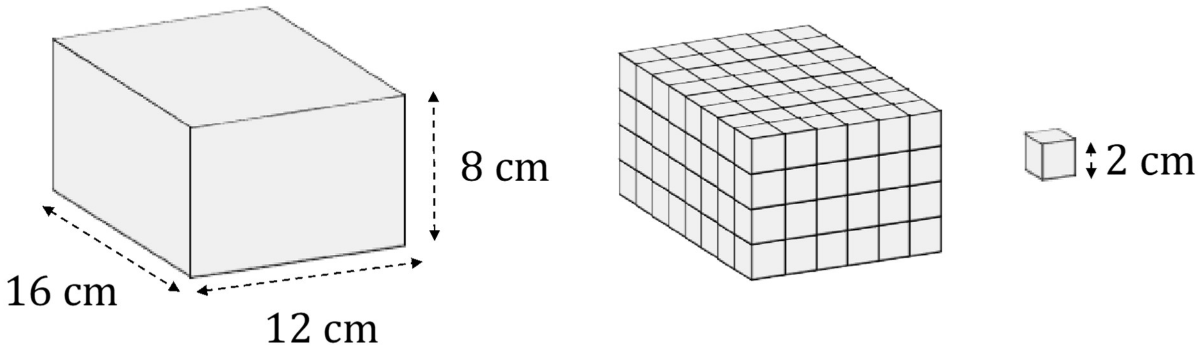
7. A pair of distinct two-digit numbers have a common factor of 16.

a) Find three possible pairs

b) Find all the possible values for the HCF.

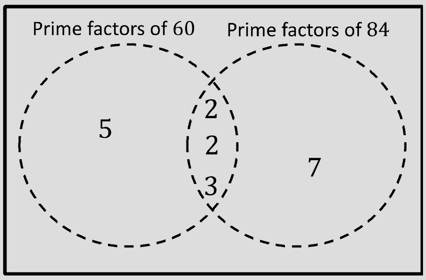
Questions for depth:

1. How else can you split the cuboid into identical cubes? Explore this for different sized cuboids.



Week 4 Session 2: More highest common factor

Concept Corner



$60 = 2 \times 2 \times 3 \times 5$
$84 = 2 \times 2 \times 3 \times 7$

60 | prime | 84 | product | 6 | 12 | HCF

Writing numbers as a _________ of their ______ factors helps to reveal common factors. The Venn diagram shows the prime factors of _____ and _____. We can see that ____ is a common factor and _____ is the ____.

1. Write each of the numbers as a product of primes:

a) 130 b) 104 c) 56 d) 308

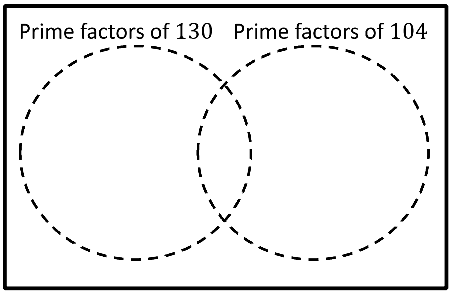
2. Given that $1680 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7$

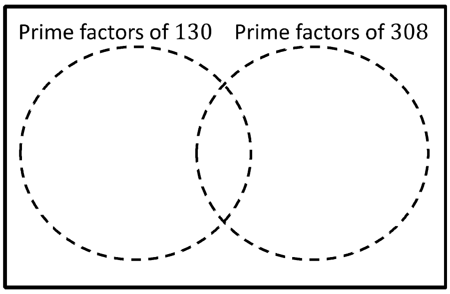
a) Decide whether each of the following are factors of 1680:

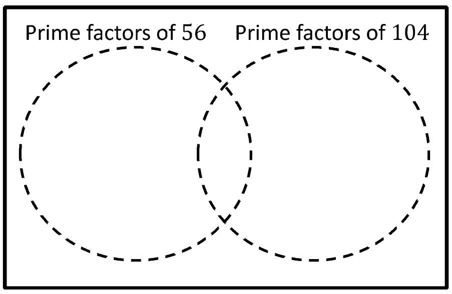
i. 2 ii. 7 iii. 14 iv. 13
v. 8 vi. 11 vii. 32 viii. 48

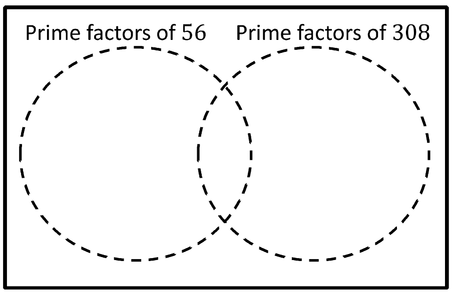
b) Which of the numbers from Q1 is a factor of 1680?

3. Copy and complete the Venn diagrams:

a)


b)


c)


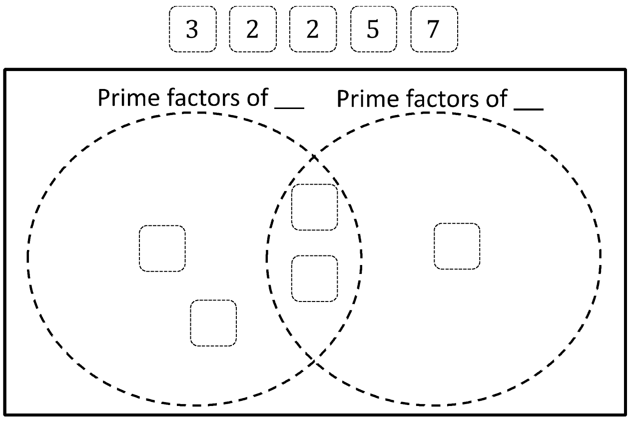
d)


4. Find the highest common factor of each pair

a) 130 and 104 b) 130 and 308 c) 56 and 104

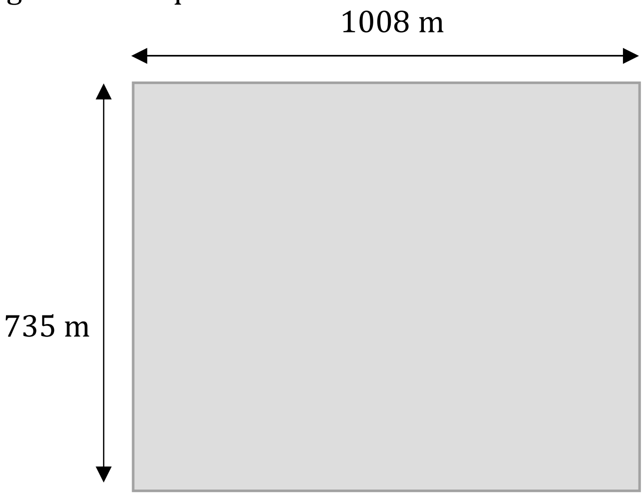
d) 56 and 308 e) 130 and 56 f) 308 and 104

5. Place the primes into the Venn diagram. Find the possible pairs of numbers and their highest common factors:



6. Find three examples of a pair of numbers greater than 1000 that have a HCF of 72.

7. A rectangular field needs to be divided in to equally sized, square plots of land. How large can the squares be?

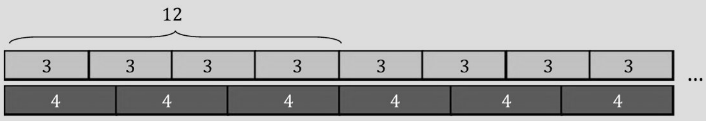


Questions for depth:

1. Compare the **HCF of $\boldsymbol{a}$ and $\boldsymbol{b}$** with the **HCF of $\boldsymbol{a}$ and $\boldsymbol{a+b}$** . Select your own values for a and b. What do you notice? Will this always be true? Explain your answer.

Week 4 Lesson 3: Lowest common multiple

Concept Corner



Multiples of 3: 3 , 6 , 9 , 12 , 15 , 18 , 21 , 24 , 27 , 30

Multiples of 4: 4 , 8 , 12 , 16 , 20 , 24 , 28 , 32 ...

3	lowest
4	multiples
12	LCM

The __________ common multiple, abbreviated as the _____, can be found by listing the ____________ of each number.

We can see that the LCM of 3 and 4 is ____.

1. List the first 12 multiples of:

a) 12 b) 9 c) 7 d) 21

2. Identify the lowest common multiple of:

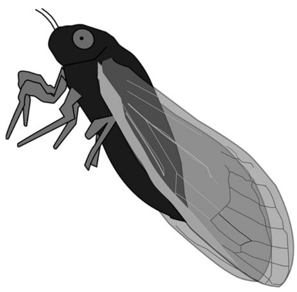
a) 12 and 9 b) 12 and 7 c) 12 and 21

d) 21 and 7 e) 21 and 9 f) 9 and 7

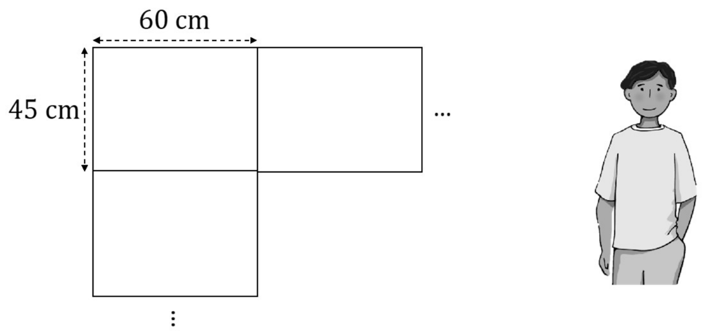
3. Find three examples of a pair of numbers that have a LCM of

a) 15 b) 21 c) 30 d) 36

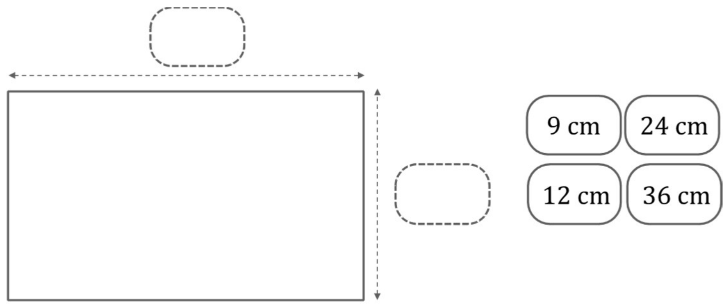
4. A cicada lives underground and appears every 17 years. A predator of the cicada appears every 4 years. If they hide at the same time, how long will it be before they appear again at the same time?



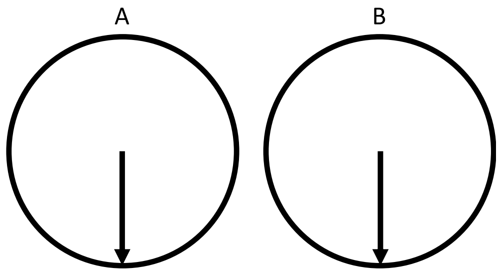
5. Charlie is trying to create a squares by tessellating the rectangle below. What size squares can he make?



6. Select two side lengths from the numbers below. Explore the different squares you can make by tessellating the rectangle:



7. **A** takes 12 seconds to do a full turn, B takes 21 seconds. They repeatedly spin in the same direction starting in the position shown:



a) How much time passes before A and B return to their starting position at the same time?

b) Will your answer change if they spin in opposite directions? If so, how?

Questions for depth:

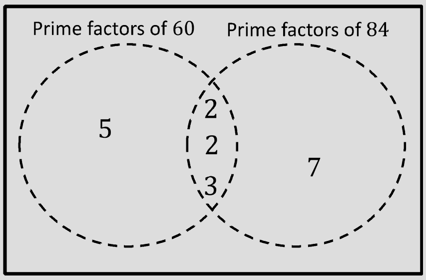
1. Following on from the situation in Q7:

a) When is the first time that the arrows point in the same direction?

b) Can the arrows both point upwards at the same time?

Week 4 Session 4: More lowest common multiple

Concept Corner



$60 = 2 \times 2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7$

84	prime
60	product
420	LCM

Writing numbers as a _________ of their ______ factors helps you to find the lowest common multiple.

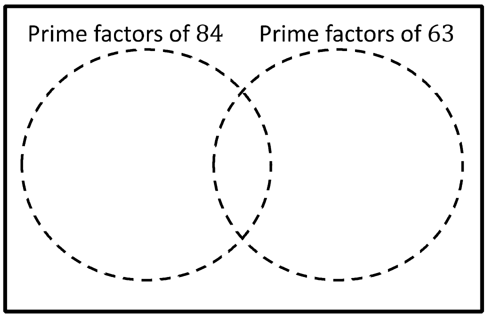
The Venn diagram shows the prime factors of 60 and 84. We can see that the _____ is ____ × 7 = ____ × 5 = ____

1. Write each of the numbers as a product of primes:

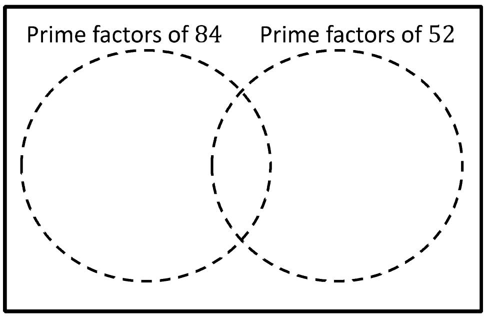
a) 63 b) 84 c) 52 d) 36

2. Copy and complete the Venn diagrams:

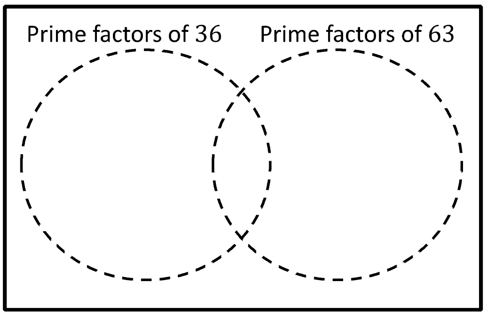
a)



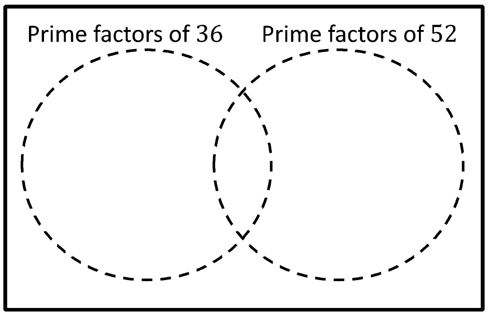
b)



c)



d)

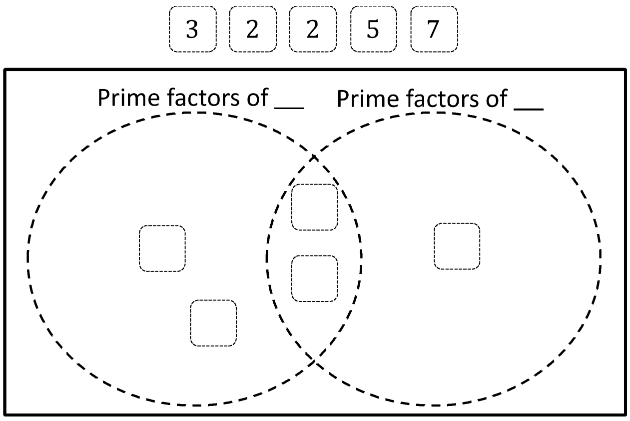


3. Find the lowest common multiple of each pair:

a) 84 and 63   b) 84 and 52   c) 36 and 63

d) 36 and 52   e) 84 and 36   f) 63 and 52

4. Place the primes into the Venn diagram. Find the possible pairs of numbers and their lowest common multiples:

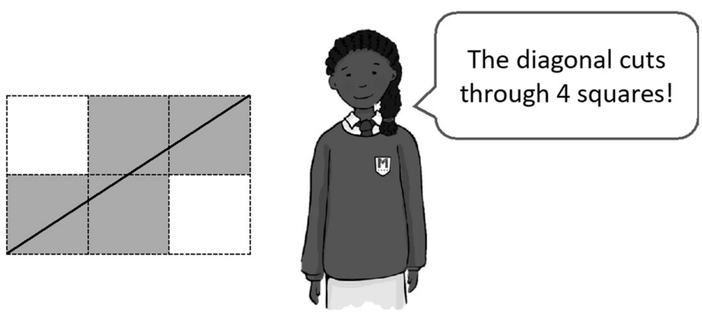


5. Find examples of pairs of two digit numbers with a LCM greater than 1000.

Questions for depth:

1. Find the product of the highest common factor and the lowest common multiple for different pairs of numbers. What do you notice? Why does this happen?

2. Neda is investigating the number of squares that a rectangle's diagonal crosses. Investigate how many squares are crossed for different rectangles. What do you notice?



Week 5: Fractions

Session 1: Part of a whole

Concept Corner

We can use fraction notation to describe part of a ________.

The _____________ represents the total number of equal parts, the _____________ represents the number of parts we are describing.

$\frac{1}{3}$	whole	numerator
$\frac{2}{3}$	denominator	
one third	two thirds	

E.g. if  is the whole.

The shaded section of  represents ____ or "______ ________".

The unshaded section of  represents ____ or "______ ________".

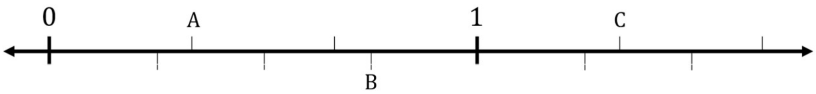
1. Find the matching pairs

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{4}{3}$ d) $\frac{3}{2}$ e) $\frac{3}{4}$

Four thirds	Two thirds	Three halves	Two fifths	Three quarters

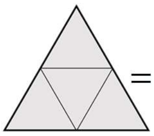
2. a) Write the values of the marked points of the number line:

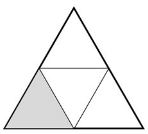
i) A =
ii) B =
iii) C =

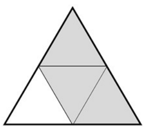


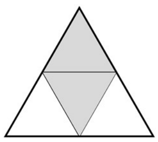
b) Suggest a value in fraction notation that lies between:

i) 0 and A: __________ ii) A and B: __________ iii) 1 and C: __________

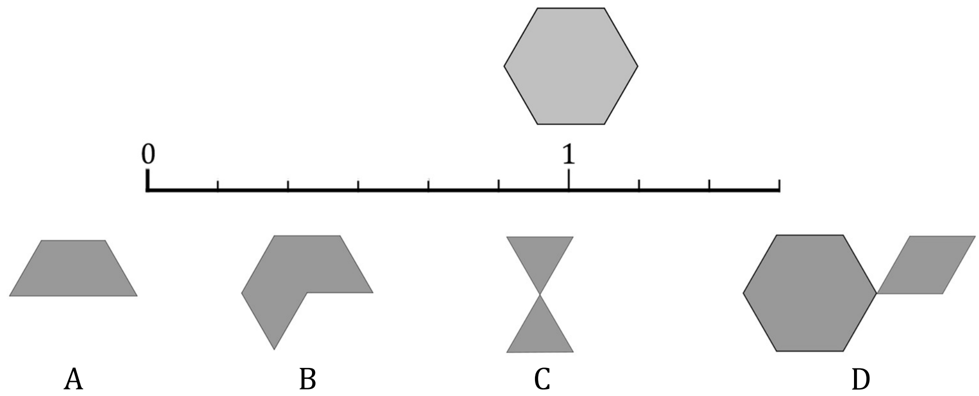
3. If  = 1 , write a fraction to represent the value of the shaded section:

a) 

b) 

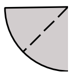
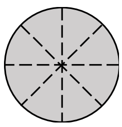
c) 

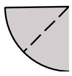
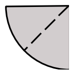
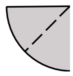
4. Draw a copy of the numberline and identify where the shapes should be placed:

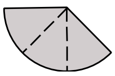
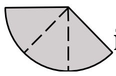
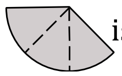


5. Sketch a diagram to complete the statement:

e.g.

 is $\frac{1}{4}$ of  ____________

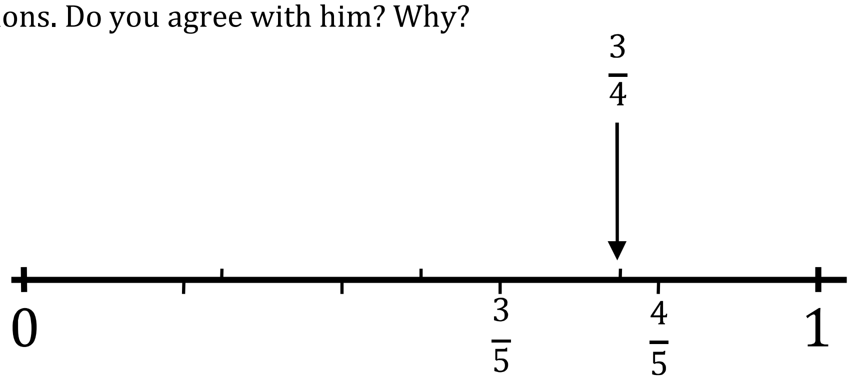
a)  is $\frac{1}{2}$ of __________ b)  is $\frac{1}{3}$ of __________ c)  is $\frac{2}{3}$ of __________

d)  is $\frac{1}{2}$ of ________ e)  is $\frac{3}{4}$ of ________ f)  is $\frac{1}{3}$ of ________

Questions for depth:

1. Tom thinks that that you will always be able to find a fraction 'in between' two other fractions. Do you agree with him? Why?

e.g.



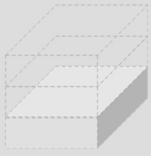
Week 5 Session 2: Fractions of measure

Concept Corner

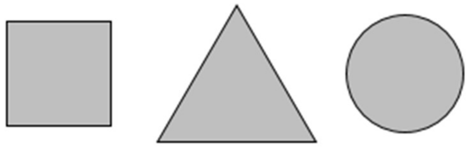
We can use _________ when describing units of measure.

Units of measure can be used to describe: _____, ______, angles, volume etc.

For example if a cube holds 1 L of water 

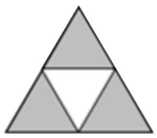
Here  we have _____ of water.

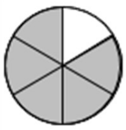
area	fractions
$\frac{1}{3}$ L	time

1. Each of the following shapes has an area of $1m^2$ 

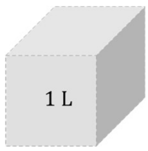
   a) Find the area of:

   i)  ____________

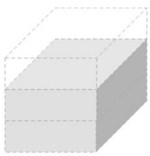
   ii)  ____________

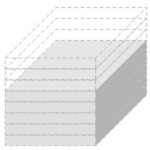
   iii)  ____________

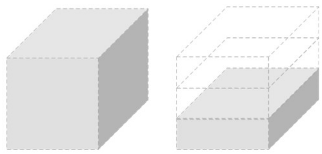
   iv)  ____________

2. A cube can hold 1L of water: 

. How much water is shown below?

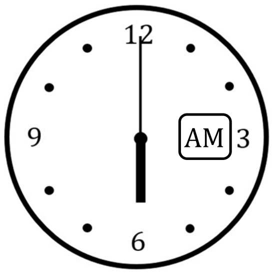
   a)  ___ L

   b)  ___ L

   c)  ___ L

3. Billy and Tommy each have 1 L water bottles, they are half full. They pour the water into a 3 L container, have fraction of the 3 L container is full?

4. This clock shows 06:00.



   a) How many minutes will have past when the minutes hand has travelled $\frac{1}{4}$ of the way around the clock?

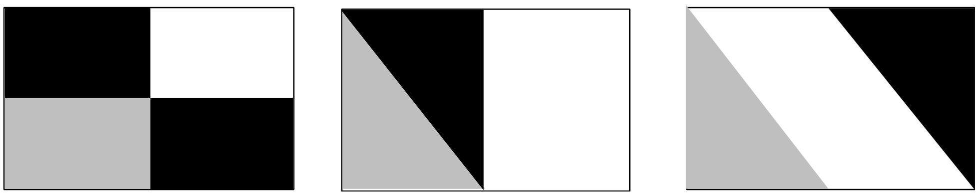
   b) What fraction of the clock will the hour hand have travelled by 10:00?

   c) What fraction of the clock will the **hour hand** have travelled when the minute hand has travelled all the way around the clock?

   d) What fraction of the clock will the **hour hand** have travelled when the minute hand has travelled half way around the clock?

5. If each flag has an area of 1m^2, find the areas of each colour in fraction notation:

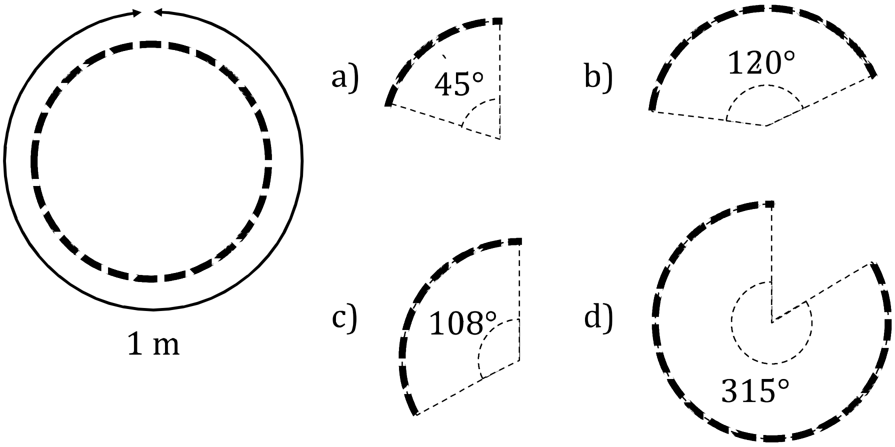
a) b) c)



6. Sketch three flags where $\frac{1}{4}$ of the area is white , $\frac{1}{4}$ is black and $\frac{1}{2}$ is grey.

Questions for depth:

1. A 1 m length of rope is made into a circle. Find the length of each section of rope:



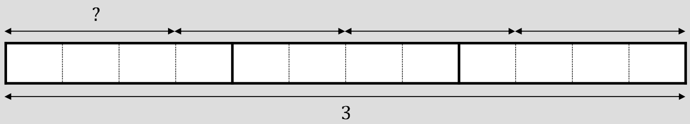
What angle is needed for a rope of length $\frac{3}{8}$ m?

Week 5 Session 3: Fair shares

Concept Corner

We can use _________ notation to describe a __________.

For example ___ ÷ ___ = ____



4

3

$\frac{3}{4}$

fraction

division

1. State the value of the shaded and unshaded sections:

a) 

b) 

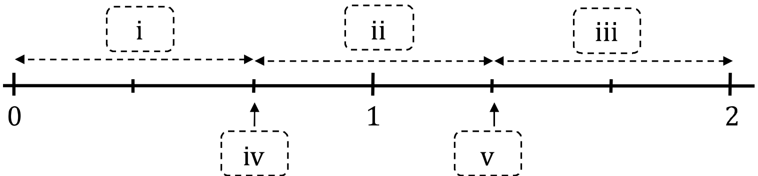
c) 

d) 

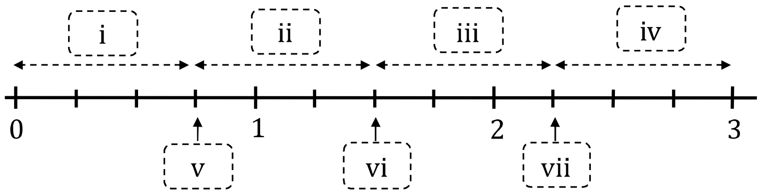
e) 

2. Find the missing numbers below:

a)



b)



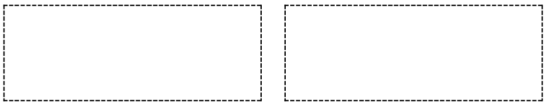
3. Write the following questions in fraction notation

a) 2 ÷ 3

b) 3 ÷ 4

c) 5 divided by 3

4. A group of friends are sharing 2 chocolate bars. What fraction of a chocolate bar do they each get if …



a) … there are 3 friends?

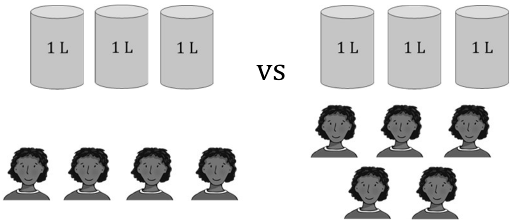
b) … there are 5 friends?

c) … there are 7 friends?

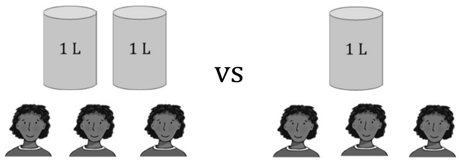
Sketch diagrams to represent each situation

5. In each situation decide which group gets more soda per person.
How much do they get each?

a)

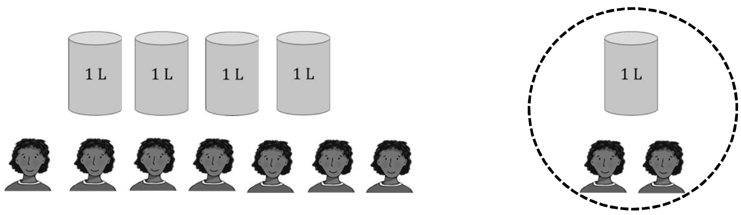


b)



c)

6. A group of 7 people plan to share 4L of soda. Two people join the party and bring 1 L of soda. Does the amount of soda per person increase or decrease?



Questions for depth:

1. n people plan to share 10 chocolate bars.

a) How much does each person get?

2 more people join the group and bring a chocolate bar with them…

b) When does the amount of chocolate per person increase? When does it decrease?

Week 5 Session 4: Equivalence

Concept Corner

Two fractions are said to be ____________ if they represent the same value.

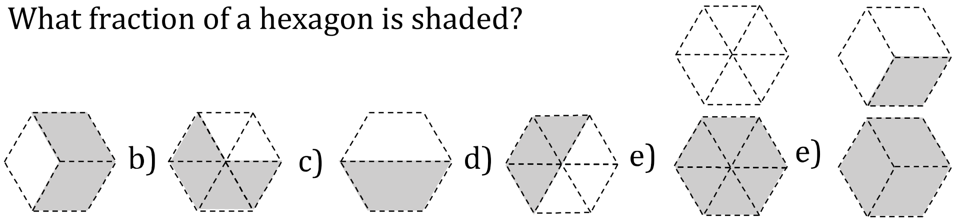
For example we can see that _____ = _____ so they are equivalent. Similarly, _____ = _____ so they are equivalent too.

$\frac{2}{3}$ $\frac{2}{6}$ $\frac{1}{3}$ $\frac{4}{6}$

equivalent

$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

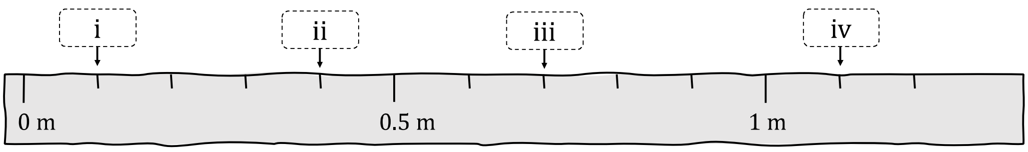
1. What fraction of a hexagon is shaded?



a) $\frac{\square}{3}$ b) $\frac{\square}{6}$ c) $\frac{\square}{\square}$ d) $\frac{3}{\square}$ e) $\frac{\square}{6}$ e) $\frac{\square}{\square}$

2.

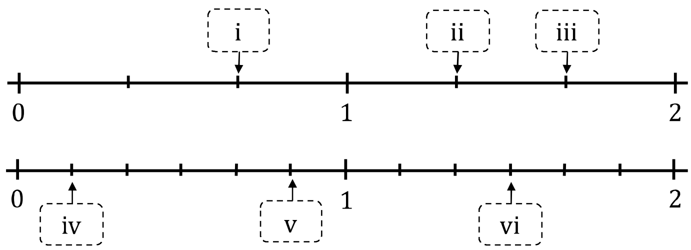
a) Find each of the marked decimal values on this tape measure:



b) Now write each of the answers in the form: $\frac{\square}{10}$

3.

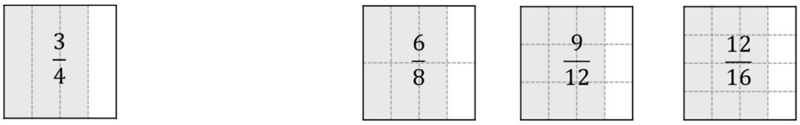
a) Find the marked numbers:



b) Use the number lines to find different ways to complete the following:

$$\frac{\square}{3} = \frac{\square}{6}$$

4. Here are three fractions that are equivalent to $\frac{3}{4}$:



Draw your own diagrams to show three fractions equivalent to: a) $\frac{2}{3}$ b) $\frac{4}{5}$ c) $\frac{4}{3}$

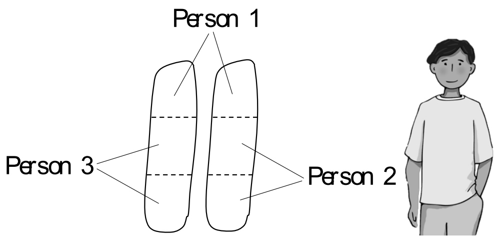
5. Copy and complete each of the following:

a) $\frac{1}{2} = \frac{\square}{10} = \square.\square$ b) $\frac{1}{5} = \frac{\square}{10} = \square.\square$ c) $\frac{2}{5} = \frac{\square}{10} = \square.\square$

d) $\frac{\square}{5} = \frac{12}{10} = \square.\square$ e) $\frac{\square}{\square} = 0.7$ f) $\frac{12}{30} = \frac{\square}{10} = \square.\square$

g) $\frac{36}{20} = \frac{\square}{10} = \square.\square$ h) $\frac{8}{25} = \frac{\square}{100} = \square.\square\square$ i) $\frac{\square}{50} = \frac{\square}{100} = 0.06$

6. Charlie has shown how he would share two sausages between three people.



a) What fraction of a sausage does each person get?

b) Draw a similar diagram to show the following.

What fraction of a sausage does each person get?

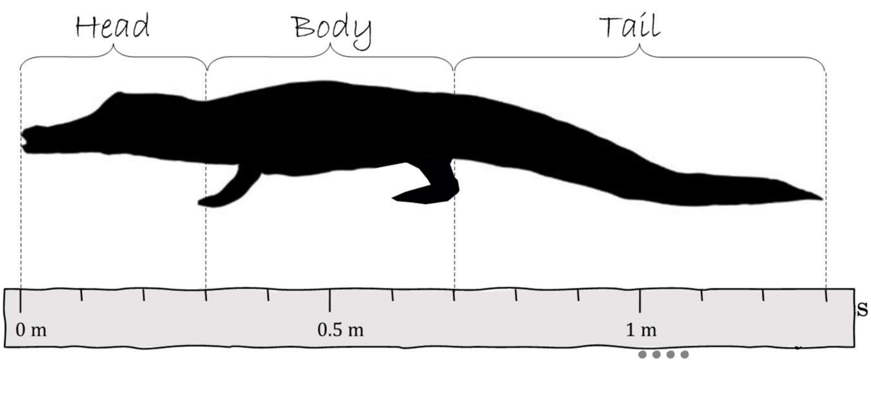
i) 4 sausages between 6 people ii) 4 sausages between 3 people iii) 8 sausages between 6 people

Questions for depth:

1.
a) What fraction of the crocodile's total length is its:

i) head? ii) body? iii) tail?

b) The world's longest recorded crocodile, Lolong, was 6.17 m long! Assuming it had the same proportions, find the approximate length of its head, body and tail.



Ark